

□ MOST POWERFUL TEST (MP TEST)

□ Goal:

To find the test that **maximizes power** (i.e., minimizes Type II error) for a **fixed significance level α** , when testing:

$$H_0: \theta = \theta_0 \text{ vs } H_1: \theta = \theta_1$$

□ Steps to Construct the Most Powerful Test:

1. Write the Likelihood Ratio (LR):

$$\Lambda(x) = \frac{f(x; \theta_1)}{f(x; \theta_0)}$$

2. Apply Neyman-Pearson Lemma:

- Reject H_0 if $\Lambda(x) > k$
- Accept H_0 if $\Lambda(x) < k$
- Randomize if $\Lambda(x) = k$ (rare in practice)

3. Determine the Critical Region:

- Choose constant k such that:

$$P_{\theta_0}(\Lambda(x) > k) = \alpha$$

4. Compute Power Function:

$$\beta(\theta_1) = P_{\theta_1}(\text{Reject } H_0)$$

5. Interpret:

- This test is **most powerful** for testing θ_0 vs θ_1

□ UNIFORMLY MOST POWERFUL TEST (UMP TEST)

□ Goal:

To find a test that is **most powerful for all values** in a **composite alternative**:

$$H_0: \theta = \theta_0 \text{ vs } H_1: \theta > \theta_0 \text{ (or } \theta < \theta_0 \text{)}$$

□ Steps to Construct the UMP Test:

1. Check for Monotone Likelihood Ratio (MLR):

- o Show that the family $\{f(x;\theta)\}$ has MLR in a statistic $T(x)$
- o That is:

$\frac{f(x;\theta_2)}{f(x;\theta_1)}$ is increasing in $T(x)$ for $\theta_2 > \theta_1$

2. Use Karlin-Rubin Theorem:

- o If MLR holds in $T(x)$, then the test:

$$\phi(x) = \begin{cases} 1 & \text{if } T(x) > c \\ 0 & \text{otherwise} \end{cases}$$

is UMP of size α

3. Determine Critical Value c :

- o Solve:

$$P_{\theta_0}(T(x) > c) = \alpha$$

4. Write the Rejection Rule:

- o Reject H_0 if $T(x) > c$

5. Interpret:

- o This test is **uniformly most powerful** for all $\theta > \theta_0$

□ Quick Recall Tips:

Concept	Key Idea	Formula
MP Test	Best test for one specific θ_1	Use LR and Neyman-Pearson
UMP Test	Best test for all $\theta > \theta_0$	Use MLR + Karlin-Rubin
MLR	Likelihood ratio increases with statistic	$\Lambda(x) \uparrow T(x)$
Critical Region	Where you reject H_0	Based on LR or $T(x) > c$